Comparison of AGE and spectral methods for the simulation of far-wakes

By D.K. Bisset AND M.M. Rogers

1. Motivation and objectives

Turbulent flow simulation methods based on finite differences are attractive for their simplicity, flexibility and efficiency, but not always for accuracy or stability. This report demonstrates that a good compromise is possible with the Advected Grid Explicit (AGE) method. AGE has proven to be both efficient and accurate for simulating turbulent free-shear flows, including planar mixing layers and planar jets. Its efficiency results from its localized fully explicit finite difference formulation (Bisset 1998a,b) that is very straightforward to compute, outweighing the need for a fairly small timestep. Also, most of the successful simulations were slightly under-resolved, and therefore they were, in effect, large-eddy simulations (LES) without a sub-grid-scale (SGS) model, rather than direct numerical simulations (DNS). The principle is that the role of the smallest scales of turbulent motion (when the Reynolds number is not too low) is to dissipate turbulent energy, and therefore they do not have to be simulated when the numerical method is inherently dissipative at its resolution limits. Such simulations are termed 'auto-LES' (LES with automatic SGS modeling) in this report.

The quality of a numerical method must be judged ultimately through comparisons with experimental results, but there is always the difficulty of not knowing the exact initial or boundary conditions in turbulent flow experiments. This problem can largely be overcome when comparing with other numerical simulations. Thus, the aims of this work are (a) to compare, in terms of accuracy and efficiency, a well-resolved AGE method DNS with a high quality spectral DNS under identical conditions, and (b) to assess the effects of reducing the resolution, that is to compare AGE method auto-LES of the same flow with the DNS results. The comparison spectral DNS is a temporally-evolving plane wake of a parallel flat plate with turbulent boundary layers simulated by Moser, Rogers & Ewing (1998), using the method of Spalart, Moser & Rogers (1991). This flow has also been simulated with spectral LES by Ghosal & Rogers (1997) from almost identical initial conditions, and it is very similar to some of the experiments carried out by Weygandt & Mehta (1995). The temporal approximation to spatial development is very good in a far-wake (the streamwise rate of change of properties is very slow), and the streamwise-periodic boundary conditions simplify the problem very significantly and also match exactly the assumption of periodicity inherent in spectral methods.

Previous AGE simulations have all concerned flows developing spatially with inflow and outflow boundary conditions, namely, two-stream mixing layers (Bisset 1998a,b,c) and a turbulent jet into still surroundings (Bisset & Antonia 1998). The present comparison using periodic boundary conditions is therefore not a complete test of the AGE method as usually applied. In fact the 'advected grid' feature is not being used at all in the numerical sense, because the mean flow of the wake has been removed from the problem by the temporal transformation, and the original fixed physical boundaries are irrelevant after the computations begin. Thus a temporal flow comparison results in a slightly less demanding test of AGE capabilities, but this is outweighed by the opportunity to compare with another method under well-defined conditions. On the other hand, the efficiency advantage of the AGE method is less significant in temporal simulations than in spatial, because adequate statistics can be obtained from one or a few realizations of a temporal simulation averaged in the streamwise and spanwise directions. The results to follow show a very satisfactory level of agreement, judging partly by contrast with the quite different results that were obtained from another AGE simulation with different initial conditions, namely a simple 'top-hat' initial velocity profile.

Much of the material herein is treated more extensively by Bisset (1999), except for the results of varying the apparent acoustic speed. Only a minimal description of the comparison spectral simulation is given; more details are available from the report by Bisset, Hunt & Rogers in the present volume and from the original references.

2. Accomplishments

2.1 Conditions for DNS

Moser, Rogers & Ewing (1998) numerically simulated the wake of a parallel flat plate with turbulent boundary layers at a Reynolds number $\dot{m}/\nu = 2000$, where $\dot{m}$ is effectively the mean velocity deficit $\langle U \rangle$ integrated over y. Angle brackets indicate averaging over any plane of constant y, and lower case letters indicate fluctuations, thus $U = \langle U \rangle + u$. The initial state was obtained by placing data from two different instants in a boundary layer simulation back-to-back (without the wall) in a periodic domain of size $50\dot{m}/U_d$ streamwise and $12.5\dot{m}/U_d$ spanwise, where U_d is the mean centerplane velocity deficit initially. The simulation was carried out for 125 time units $\tau = t\, {U_d}^2/\dot{m}$ with a variable timestep Galerkin incompressible spectral method using up to $512 \times 195 \times 128$ modes (adjusted several times during the total run). From $\tau = 40$ the mean velocity profile was self-similar when normalized by the centerplane velocity defect U_0 and the half-velocity width b, and the growth rate $(1/U_0)db/dt$ was constant, but by $\tau = 100$ the spanwise correlation length was becoming too large for the domain, and self-similarity broke down. Results will be shown near the beginning ($\tau = 42.8$) and especially near the end ($\tau = 91.5$) of this range. Note that b is taken from the full $\langle U \rangle$ profile, not the half profile as used by some authors.

For the AGE simulation, spectral vorticity data from $\tau = 1.4$ were converted to velocity components on a $514 \times 320 \times 130$ physical grid with $\Delta x = \Delta z = 1.95\Delta y$, and ten-point-thick damping layers at the top and the bottom. The AGE method does not assume precisely incompressible flow, and therefore an initial pressure field was calculated separately with a Poisson equation. The velocity of pressure waves (acoustic velocity c) should be kept reasonably low, because the timestep size limit is inversely proportional. It was adjusted so that U_0 was about $0.3c$ initially; U_0 decreased through the run to finish at less than $0.08c$, which means that the flow was effectively incompressible in terms of usual laboratory practice. A few other values of c were tried also. The timestep was set to $\Delta\tau = 0.02$, the acoustic maximum based on Δy. Parameters W_P (pressure smoothing) and W_V (targeted diffusion) were varied, as detailed by Bisset (1999). Both spectral and AGE calculations included a passive scalar, but results will not be considered here.

Another temporal wake was simulated as a preliminary test, mainly to 'calibrate' the scale of any differences between AGE and spectral results in the main comparison. The spectral simulation results for Reynolds stresses, for example, were significantly lower than those of quite a number of self-similar far-wakes, and it was not clear in advance how well the AGE method simulation would respond to the strong boundary layer effects in the initial velocity field. The preliminary simulation began from a simple top-hat velocity profile with a small amount of random noise superimposed along the planes of the velocity discontinuity, rather like the wake of a screen or porous plate normal to the flow. The mean velocity profile was approximately self-similar from $\tau = 10$ onwards. Self-similarity was achieved sooner in this case at least in part because of the normalization with U_d (no cusp in the mean velocity profile in this wake). Normalized Reynolds stress profiles were also self-similar over much of the period, and they were much closer to the experimental results (Weygandt & Mehta 1995, or Figure 5 of Moser, Rogers & Ewing 1998) than were the main simulation results. Most importantly in this context, the differences between the top-hat wake and the wake from turbulent boundary layers turned out to be vastly greater than the differences between the AGE and spectral versions of the latter.

2.2 Conditions for auto-LES

The starting fields for the auto-LES runs were obtained by simply omitting data points (one out of two, two out of three, or seven out of eight, in each direction) from an instant in the AGE DNS calculation. The reduced grids are denoted R2, R3 and R8 respectively. At $\tau = 1.4$, the original starting point, the cusp in the mean velocity profile was too sharp to be represented accurately in the reduced data, so the instant $\tau = 10.4$ was chosen. Here the mean velocity profile is still far from its self-similar form, but the peak value has dropped by more than 50% and the cusp has begun to spread out. The timestep was increased to the acoustic limit for each level of data reduction. Upper and lower damping layers were maintained at ten points thick, so the R8 grid was $66 \times 58 \times 18$ points. More than 99.8% of the original undamped data points were eliminated in this case. No changes were made to the AGE method algorithm for the auto-LES runs.

2.3 AGE DNS results

AGE parameter values for all DNS results in this section were $W_V = 1.5$ and $W_P = 0.5$, but the parameter values for DNS are not at all critical (Bisset 1999). Spectral and AGE DNS agree very well about growth of the wake width b, while U_0 tends to decay slightly more quickly in the AGE simulations, and at $\tau = 91.5$ it is low

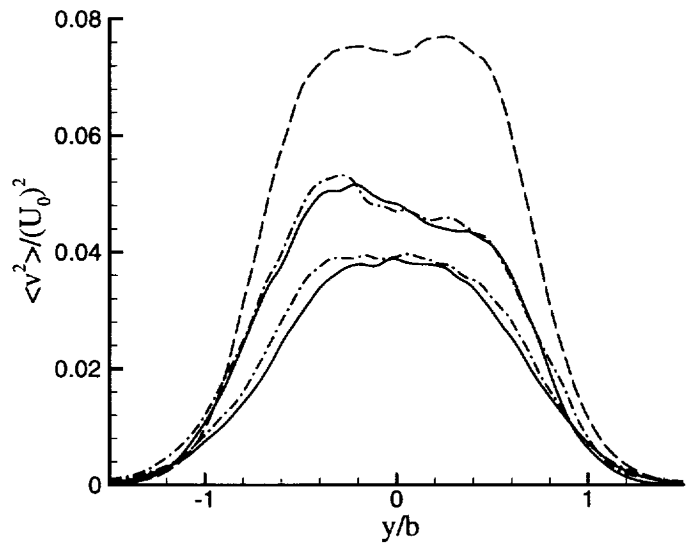

FIGURE 1. Distributions of $\langle v^2\rangle$ from spectral (– – –) and AGE (———) DNS at $\tau = 42.8$ (lowest curves) and $\tau = 91.5$ (middle curves). The uppermost curve is $\langle v^2\rangle$ at $\tau = 74.4$ from the top-hat initial profile.

by about 3%. Normalized mean velocity distributions from $\tau = 40$ onwards collapse very well onto the spectral self-similar results. While the slightly faster decay of U_0 is undesirable, it should be viewed from the perspective that the growth rates (in b) of most far-wakes are of order 50% larger, as discussed by Moser, Rogers & Ewing (1998), and therefore the AGE simulation has retained virtually all of the effects of the turbulent boundary layer initial conditions in this case.

The effects of the two different boundary layer realizations on the two sides of the wake are actually somewhat different, which appears most clearly in profiles of $\langle v^2\rangle$, the variance of the transverse velocity fluctuations, shown in Figure 1. Compared to free shear flows, boundary layers tend to have relatively less energy in the transverse and spanwise fluctuations, and shorter spanwise length scales, so it is not surprising that $\langle v^2\rangle$ increases during the earlier stages of this simulation, nor that the spanwise correlation length becomes the limiting factor in a domain where the width originally suited a boundary layer. The continuing slow increase in $\langle v^2\rangle$ throughout the self-similar period (Figure 1), in contrast to profiles of other quantities, is analyzed by Moser, Rogers & Ewing (1998). The increase in $\langle v^2\rangle$ is a little faster on the lower side of the wake, giving a rather asymmetrical profile at the later time in Figure 1. Clearly the results for the two methods are very similar. Agreement for other profiles (not shown) such as $\langle uv\rangle$ is equally good, except that the spectral DNS $\langle u^2\rangle$ profile is more asymmetrical than the AGE version at the final time. The close agreement in Figure 1 for the same initial conditions may be contrasted with the results obtained from other initial conditions, i.e. the top-hat profile, also shown in Figure 1 at an intermediate time.

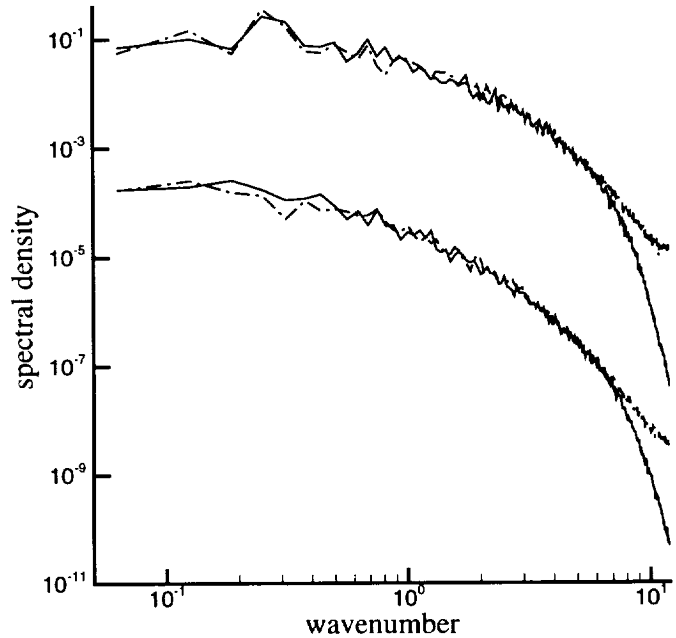

FIGURE 2. Streamwise power spectra of u (lower curves) and v (upper curves) from AGE DNS (———) and spectral DNS (—-—) at $\tau = 91.5$ on the centerplane.

Figure 2 compares streamwise power spectra of the fluctuations u and v from the centerplane at $\tau = 91.5$ from both DNS methods. Except at very high wavenumbers, the agreement is remarkably good, and where they begin to diverge the levels of the spectra have already decreased by three to four orders of magnitude. Most likely the second-order central differencing used with AGE is too diffusive at very small scales (and high wavenumbers), but it is also likely that some residual noise has raised the spectral DNS values slightly at high wavenumbers, especially for the v spectrum.

2.4 Results for auto-LES

The growth of b and decay of U_0 on the auto-LES grids deviate only slightly from the DNS results (Bisset 1999), which is significant considering that at $\tau = 10.4$ (when the projection onto these grids was made) more than 50% of the change in mean velocity across the wake occurs within two R8 grid points either side of the centerplane. Also, the rate of change of b is about right even though values of b are a bit low (a consequence of poor representation of the velocity defect initially). The value of the targeted diffusion parameter W_V was increased to 2.2 for grid R8 (see below). W_V has a significantly greater effect on auto-LES results than on DNS.

The effect of reduced resolution on $\langle v^2 \rangle$ is shown at $\tau = 91.5$ in Figure 3. The curve for the R2 case deviates a little from the DNS, and for R3 the values are slightly large on the upper side. For R8, the distribution is somewhat narrower

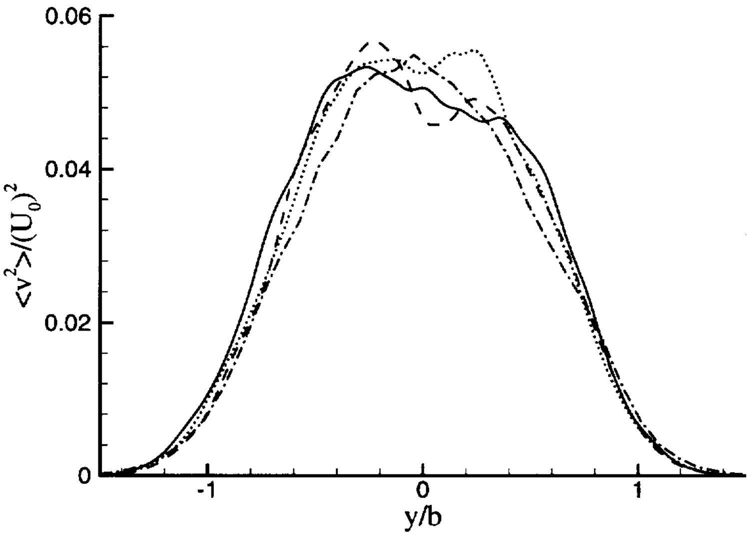

FIGURE 3. Distributions of transverse velocity variance at $\tau = 91.5$ for AGE DNS (———) and for auto-LES on R2 (- - - -), R3 (········) and R8 (— — —) grids.

overall, even though it has been normalized by the smallest b value. Roughly the same level of agreement was found for other variances and for Reynolds shear stress (not shown). Clearly the reduction in resolution with auto-LES is not degrading the results in any serious way, and the missing small scales do not contribute a great deal to the total turbulent energy.

Power spectra for velocity components u and v are presented in Figure 4, along with the corresponding pressure spectra. At the high wavenumber end, reduced resolution causes a progressively earlier rolloff in the velocity spectra, but values before the rolloff points are remarkably consistent. As mentioned above, W_V was increased to 2.2 for grid R8 in order to match the spectra (values were somewhat high at $W_V = 1.5$), but no other changes were made. Interestingly the pressure spectra do not exhibit early rolloff, although the overall level on R8 is affected by W_V just as for velocities. The most likely reason is that the pressure evolution equations do not include advection terms (see Bisset 1999).

CPU times for AGE DNS and the three levels of auto-LES (the latter scaled to include the missing time before the transfer from the DNS grid at $\tau = 10.4$) were 11.5 hours, 52 minutes (grid R2), 13 minutes (R3), and less than 40 seconds (R8). The ideal speedup goes as the fourth power of grid spacing (the timestep is enlarged along with the grid), but vector overheads and the fixed thickness damping layers reduce the actual speedup somewhat. Corresponding CPU times for spectral DNS and for the LES by Ghosal & Rogers (1997) with a dynamic SGS model were more than an order of magnitude greater.

2.5 Effects of compressibility

Since CPU time can be reduced in proportion to sonic speed, it is important

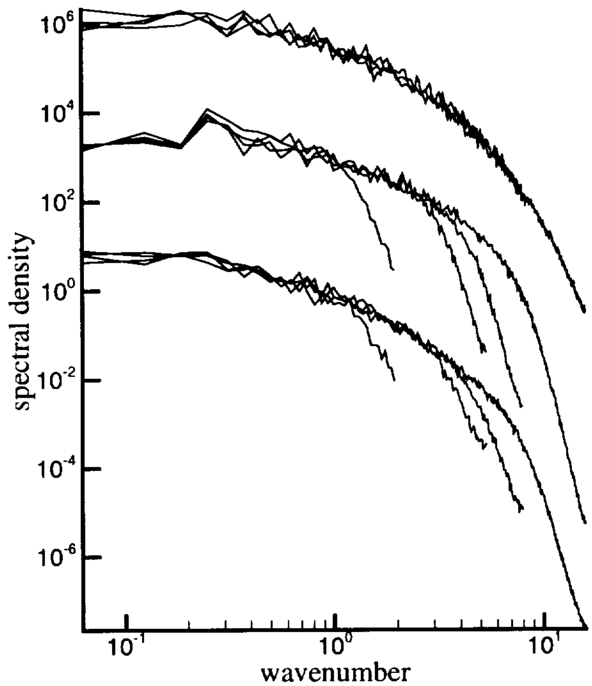

FIGURE 4. Streamwise power spectra on the centerplane at $\tau = 91.5$ for u (bottom), v (middle) and pressure (top) from AGE DNS and auto-LES. From left to right by rolloff point, the grids used were R8, R3, R2 and DNS.

to know how strongly changes in c affect the results. The effects were studied systematically on the R3 grid, but one run was made at DNS resolution (starting at $\tau = 10.4$) with c halved and Δt doubled. The largest effect was a 1.3% decrease in U_0 by $\tau = 91.5$; other quantities tended to decrease by smaller amounts before normalization, and thus some of the normalized statistics increased very slightly.

On the R3 grid, doubling c and halving Δt caused a 1.6% increase in U_0, with smaller effects on other quantities. However, restoring c (but not Δt) had very little effect, showing that Δt is at least as important as c, where c is not too small. Reducing c and increasing Δt by factors of 2.0 or 4.0 caused U_0 to drop by 2% or 4%, and other statistics decreased a little; spectra showed more high-wavenumber rolloff. Reducing c without changing Δt produced small *increases* in U_0 while other quantities still decreased. Note that $0.7 > U_0/c > 0.3$ during the runs with c reduced by 4.0, and true physical effects would be expected in this range. These tests seem to confirm the original concept that U_0/c (and therefore Δt) are not critical if they are kept fairly small, but they show somewhat greater sensitivity to Δt than found previously (Bisset 1998a).

3. Future plans

Several of the more significant numerical aspects of the AGE method as applied to free-shear flows have already been investigated and analyzed by Bisset (1999), so the main area of work at present is the development of the AGE method for wall-bounded flows, again with comparisons to spectral DNS. Preliminary results in a planar channel are reasonable, but they imply that AGE will not show the major efficiency advantage over spectral methods that it has in free-shear flows. Nevertheless, its simplicity and flexibility in dealing with boundary conditions are also important advantages, and it is likely that compressible subsonic flow can also be simulated. While the auto-LES approach should work well in the central/outer regions of bounded flows, it remains to be shown that it works in the vicinity of walls.

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